匀速运动点电荷的电磁场的波动性

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摘　要：通过对匀速运动点电荷的电磁场场强方程求解二阶偏导数，发现匀速运动点电荷的电磁场场强方程符合波动方程特征，并得到匀速运动点电荷的电磁场场强的波速.

关键词：电磁场　匀速运动　点电荷

点电荷激发的静电场表达式只是空间变量的函数，静电场的分布不随时间变化，静电场的传播并非是超距传播，而是静电场处于平衡状态. 静电场的传播速度直接给出是光速，传播方式却没有进一步说明，其实静电场的传播方式和匀速运动电磁场的传播方式是相似的，在对匀速运动的点电荷的电磁场场强方程求解二阶偏导数时，发现匀速运动点电荷激发的电磁场方程符合波动方程特征.

1　匀速运动点电荷电磁场方程的二阶偏导数

真空状态，在空间直角坐标系中，点电荷 q 在 $t=0$ 时从原点出发，沿 x 轴正方向以速率 v 做匀速直线运[1,2]，t 时刻位于 x 轴上的点 $A(vt,0,0)$，$\boldsymbol{r}=(x-vt)\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k}$ 是点电荷 q 到场点 $P(x,y,z)$ 的矢径，矢径 $\boldsymbol{r}$ 与电荷运动速度 $\boldsymbol{v}$ 之间的夹角为 θ，如图 1 所示.

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Exploring the Sublimation Way of Classroom Knowledge from the Intersection of Marxist Philosophy and Physics Curriculum Contents

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Abstract: Physics and philosophy have the same origin. Taking advantage of the ideological and political reform of the curriculum, the author focused on the connection and intersection between Marxist philosophy and the content of the "University Physics Course" in terms of world view, scientific method and concept of history. This helps to explore ways to improve physics teaching Ideological and methodological connotations, and possible sublimation paths for the theoretical content of classroom teaching.

Key words: college physics; Marxist philosophy; scientific method; history and logic

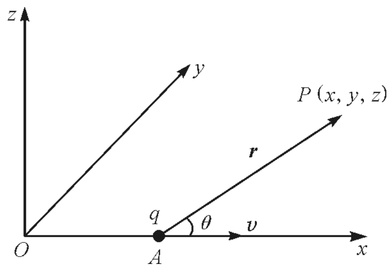

图 1 沿 x 轴正向匀速运动的点电荷

由电动力学可知，沿 x 轴正向以速率 v 匀速运动的点电荷 q，其产生的电场和磁场可表示为

$$\boldsymbol{E}=\frac{q(1-\beta^2)}{4\pi\varepsilon_0 r^3\ (1-\beta^2\ \sin^2\theta)^{\frac{3}{2}}}\boldsymbol{r} \tag{1}$$

$$\boldsymbol{B}=\frac{1}{c^2}\boldsymbol{v}\times\boldsymbol{E} \tag{2}$$

式中 $\beta=\frac{v}{c}$，c 为真空中的光速．

式(1) 中场强 $\boldsymbol{E}$ 可分解为 3 个坐标轴方向的分矢量

$$\boldsymbol{E}_x=\frac{q(1-\beta^2)(x-vt)}{4\pi\varepsilon_0[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{\frac{3}{2}}}\boldsymbol{i} \tag{3}$$

$$\boldsymbol{E}_y=\frac{q(1-\beta^2)y}{4\pi\varepsilon_0[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{\frac{3}{2}}}\boldsymbol{j} \tag{4}$$

$$\boldsymbol{E}_z=\frac{q(1-\beta^2)z}{4\pi\varepsilon_0[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{\frac{3}{2}}}\boldsymbol{k} \tag{5}$$

对式(3) 分别求 x,y,z,t 的二阶偏导数得

$$\frac{\partial^2\boldsymbol{E}_x}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial\boldsymbol{E}_x}{\partial x}\right)=\frac{\partial}{\partial x}\left\{\frac{q(1-\beta^2)[-2(x-vt)^2+(1-\beta^2)(y^2+z^2)]}{4\pi\varepsilon_0[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{\frac{5}{2}}}\right\}\boldsymbol{i}=$$
$$\frac{3q(1-\beta^2)(x-vt)[2(x-vt)^2-3(1-\beta^2)(y^2+z^2)]}{4\pi\varepsilon_0[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{\frac{7}{2}}}\boldsymbol{i} \tag{6}$$

$$\frac{\partial^2\boldsymbol{E}_x}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial\boldsymbol{E}_x}{\partial y}\right)=\frac{\partial}{\partial y}\left\{\frac{-3q(1-\beta^2)^2(x-vt)y}{4\pi\varepsilon_0[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{\frac{5}{2}}}\right\}\boldsymbol{i}=$$
$$\frac{-3q(1-\beta^2)^2(x-vt)[(x-vt)^2+(1-\beta^2)(-4y^2+z^2)]}{4\pi\varepsilon_0[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{\frac{7}{2}}}\boldsymbol{i} \tag{7}$$

$$\frac{\partial^2\boldsymbol{E}_x}{\partial z^2}=\frac{\partial}{\partial z}\left(\frac{\partial\boldsymbol{E}_x}{\partial z}\right)=\frac{\partial}{\partial z}\left\{\frac{-3q(1-\beta^2)^2(x-vt)z}{4\pi\varepsilon_0[(x-vt)^2+y^2+z^2]^{\frac{5}{2}}}\right\}\boldsymbol{i}=$$
$$\frac{-3q(1-\beta^2)^2(x-vt)[(x-vt)^2+(1-\beta^2)(y^2-4z^2)]}{4\pi\varepsilon_0[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{\frac{7}{2}}}\boldsymbol{i} \tag{8}$$

$$\frac{\partial^2\boldsymbol{E}_x}{\partial t^2}=\frac{\partial}{\partial t}\left(\frac{\partial\boldsymbol{E}_x}{\partial t}\right)=\frac{\partial}{\partial t}\left\{\frac{-vq(1-\beta^2)[-2(x-vt)^2+(1-\beta^2)(y^2+z^2)]}{4\pi\varepsilon_0[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{\frac{5}{2}}}\right\}\boldsymbol{i}=$$
$$\frac{3qv^2(1-\beta^2)(x-vt)[2(x-vt)^2-3(1-\beta^2)(y^2+z^2)]}{4\pi\varepsilon_0[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{\frac{7}{2}}}\boldsymbol{i} \tag{9}$$

$$\frac{\partial^2\boldsymbol{E}_x}{\partial x^2}+\frac{\partial^2\boldsymbol{E}_x}{\partial y^2}+\frac{\partial^2\boldsymbol{E}_x}{\partial z^2}=\frac{3q\beta^2(1-\beta^2)(x-vt)[2(x-vt)^2-3(1-\beta^2)(y^2+z^2)]}{4\pi\varepsilon_0[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{\frac{7}{2}}}\boldsymbol{i}=\frac{1}{c^2}\frac{\partial^2\boldsymbol{E}_x}{\partial t^2} \tag{10}$$

对式(4) 分别求 x,y,z,t 的二阶偏导数得

$$\frac{\partial^2\boldsymbol{E}_y}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial\boldsymbol{E}_y}{\partial x}\right)=\frac{\partial}{\partial x}\left\{\frac{-3q(1-\beta^2)(x-vt)y}{4\pi\varepsilon_0[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{\frac{5}{2}}}\right\}\boldsymbol{j}=$$
$$\frac{3q(1-\beta^2)y[4(x-vt)^2-(1-\beta^2)(y^2+z^2)]}{4\pi\varepsilon_0[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{\frac{7}{2}}}\boldsymbol{j} \tag{11}$$

$$\frac{\partial^2\boldsymbol{E}_y}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial\boldsymbol{E}_y}{\partial y}\right)=\frac{\partial}{\partial y}\left\{\frac{q(1-\beta^2)[(x-vt)^2+(1-\beta^2)(-2y^2+z^2)]}{4\pi\varepsilon_0[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{\frac{5}{2}}}\right\}\boldsymbol{j}=$$

$$\frac{-3q\ (1-\beta^2)^2 y[3\ (x-vt)^2+(1-\beta^2)\ (-2y^2+3z^2)]}{4\pi\varepsilon_0\ [(x-vt)^2+(1-\beta^2)\ (y^2+z^2)]^{\frac{7}{2}}}\boldsymbol{j} \tag{12}$$

$$\frac{\partial^2 \boldsymbol{E}_y}{\partial z^2}=\frac{\partial}{\partial z}\left(\frac{\partial \boldsymbol{E}_y}{\partial z}\right)=\frac{\partial}{\partial z}\left\{\frac{-3q\ (1-\beta^2)^2 yz}{4\pi\varepsilon_0\ [(x-vt)^2+(1-\beta^2)\ (y^2+z^2)]^{\frac{5}{2}}}\right\}\boldsymbol{j}=$$

$$\frac{-3q\ (1-\beta^2)^2 y[(x-vt)^2+(1-\beta^2)\ (y^2-4z^2)]}{4\pi\varepsilon_0\ [(x-vt)^2+(1-\beta^2)\ (y^2+z^2)]^{\frac{7}{2}}}\boldsymbol{j} \tag{13}$$

$$\frac{\partial^2 \boldsymbol{E}_y}{\partial t^2}=\frac{\partial}{\partial t}\left(\frac{\partial \boldsymbol{E}_y}{\partial t}\right)=\frac{\partial}{\partial t}\left\{\frac{3qv\ (1-\beta^2)\ (x-vt)\ y}{4\pi\varepsilon_0\ [(x-vt)^2+(1-\beta^2)\ (y^2+z^2)]^{\frac{5}{2}}}\right\}\boldsymbol{j}=$$

$$\frac{3qv^2\ (1-\beta^2)\ y[4\ (x-vt)^2-(1-\beta^2)\ (y^2+z^2)]}{4\pi\varepsilon_0\ [(x-vt)^2+(1-\beta^2)\ (y^2+z^2)]^{\frac{7}{2}}}\boldsymbol{j} \tag{14}$$

$$\frac{\partial^2 \boldsymbol{E}_y}{\partial x^2}+\frac{\partial^2 \boldsymbol{E}_y}{\partial y^2}+\frac{\partial^2 \boldsymbol{E}_y}{\partial z^2}=\frac{3q\beta^2\ (1-\beta^2)\ y[4\ (x-vt)^2-(1-\beta^2)\ (y^2+z^2)]}{4\pi\varepsilon_0\ [(x-vt)^2+(1-\beta^2)\ (y^2+z^2)]^{\frac{7}{2}}}\boldsymbol{j}=\frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}_y}{\partial t^2} \tag{15}$$

对式(5)分别求 x,y,z,t 的二阶偏导数得

$$\frac{\partial^2 \boldsymbol{E}_z}{\partial x^2}+\frac{\partial^2 \boldsymbol{E}_z}{\partial y^2}+\frac{\partial^2 \boldsymbol{E}_z}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}_z}{\partial t^2} \tag{16}$$

$$\boldsymbol{E}_x+\boldsymbol{E}_y+\boldsymbol{E}_z=\boldsymbol{E} \tag{17}$$

因此式(10)、(15)、(16)合并为

$$\frac{\partial^2 \boldsymbol{E}}{\partial x^2}+\frac{\partial^2 \boldsymbol{E}}{\partial y^2}+\frac{\partial^2 \boldsymbol{E}}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} \tag{18}$$

同理可得

$$\frac{\partial^2 \boldsymbol{B}}{\partial x^2}+\frac{\partial^2 \boldsymbol{B}}{\partial y^2}+\frac{\partial^2 \boldsymbol{B}}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2} \tag{19}$$

式(18)和(19)为电场和磁场的波动方程,真空中匀速运动的点电荷的电磁场符合波动方程特征,波的相速度为光速 c.

2 结束语

匀速运动的点电荷的电磁场方程符合波动方程特征并不是计算错误,如果否定它的波动性,需要对波动的判定标准重新认识.

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Fluctuation of Electromagnetic Field of Uniform Motion Point Charge

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Abstract: By solving the second partial derivative of the electromagnetic field intensity equation of uniform moving point charge, it is found that the electromagnetic field intensity equation of uniform moving point charge conforms to the wave equation, and the wave velocity of electromagnetic field with uniform moving point charge is obtained.

Key words: electromagnetic field; uniform motion; point charge